Solutions – Exam 2 Fall 2013

1.

(a) There are two pure-strategy Nash Equilibria, (B, Y) and (A, Z). In the mixed-strategy NE, both players need to be indifferent between the pure strategies strategies which they mix. Strictly dominated strategies are never played with positive probability. Since C and X are strictly dominated, they will never be played in equilibrium and the players only mix between A and B or Y and Z. Let p be the probability that 1 plays A and q be the probability that 2 plays Y. Then player 1 is indifferent if

$$\begin{aligned} q + 3 - 3q &= 2q + 2 - 2q \\ 2q &= 2 \\ q &= \frac{1}{2} \end{aligned}$$

and player 2 is indifferent if

$$\begin{aligned} 3p + 2 - 2p &= 4p + 1 - p \\ 2p &= 1 \\ p &= \frac{1}{2} \end{aligned}$$

So the mixed-strategy NE is $(p^*, q^*) = \left(\frac{1}{2}, \frac{1}{2}\right)$ or $\left(\left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{1}{2}\right)\right)$.

(b) Player 1 has a payoff of 3 from (A, Z) and a payoff of 2 from (B, Y). From the mixed-strategy NE, 1's expected payoff is 2. He therefore gets the highest expected payoff from the pure-strategy NE (A, Z). [The answer is not complete if the expected payoff from the mixed-strategy NE is not calculated.]

(c) X is strictly dominated by Z, and C is strictly dominated by A. Both pure-strategy NE survive IESDS. It is not possible that a NE does not survive IESDS. That would mean that one of the strategies played in the NE is strictly dominated, but by definition all strategies played in a NE are optimal responses to each other. A strategy can not be both an optimal response to another strategy and strictly dominated.

2.

(a) The core is empty, since there is no action of the grand coalition (no distribution of the money among the three players) so that no two players can be made better off by sharing the money among themselves.

(b) The core is non-empty, it has two elements: (a) Asger and Christoffer get together and sell their pair of shoes for \$1, and Christoffer gets the money. (b) Bo and Christoffer get together and do the same. In both cases, there is no possible "break-out" coalition which could make a pareto-improvement.

3.

(a) The firm's profit functions are

$$\begin{aligned}\pi_1(q_1, q_2) &= (a - q_1 - q_2 - c_1)q_1 \\ \pi_2(q_1, q_2) &= (a - q_1 - q_2 - c_2)q_2\end{aligned}$$

which, after taking the first-order condition, gives us the best-response functions

$$\begin{aligned}q_1 &= \frac{a - q_2 - c_1}{2} \\ q_2 &= \frac{a - q_1 - c_2}{2}\end{aligned}$$

and the equilibrium quantities

$$\begin{aligned}q_1^* &= \frac{a + c_2 - 2c_1}{3} \\ q_2^* &= \frac{a + c_1 - 2c_2}{3}\end{aligned}$$

(b) Overall quantity is

$$q^* = q_1^* + q_2^* = \frac{2a - c_1 - c_2}{3}$$

If c_1 grows, q^* falls. If the cost of one of the firms falls, that firm will produce less. The other firm will then be able to produce a bit more to get a larger share of the market – but optimally, not as much more as the other firm is producing less, because that would lower the price too much.

(c) If $c_1 < c_2$, then also $c_1 - 2c_2 < c_2 - 2c_1$, which means that $q_1^* > q_2^*$. The firm with the lower cost can produce a higher quantity in equilibrium, because for any given price its profit per unit is higher.

4.

(a) It is a dynamic game of imperfect information.

(b) There is one subgame. The strategy sets of the players are $S_1 = \{UA, UB, DA, DB\}$ and $S_2 = \{L, R\}$.

(c) The unique SPNE is (UB, R).

(d) The bimatrix is:

	L	R
UA	$2,4$	$2,4$
UB	$2,4$	$2,4$
DA	$4,4$	$0,5$
DB	$5,0$	$1,1$

(UA, R) is a NE of the game, but not a SPNE, because (A, R) is not an equilibrium of the subgame. It can never be the case that a SPNE is not a NE because the SPNE concept is a refinement of the NE concept, i.e. every SPNE is a NE that has also passed the subgame-perfection test.

(e) The new strategy set of player 1 is $S_1 = \{UAA, UAB, UBA, UBB, DAA, DAB, DBA, DBB\}$. The SPNE of this modified game is (UBB, R).

5.

(a) The two PBE are $\left((L, L), (u, d), p = \frac{1}{2}, q \geq \frac{1}{3}\right)$ and $((L, R), (u, u), p = 1, q = 0)$.

(b) Signaling requirement 6 says that the receiver does not believe that equilibrium-dominated messages are sent with positive probability. In this case, requirement 6 therefore demands that in the pooling PBE where the sender plays (L, L) and the receiver plays $(u, d,)$, q should be 0. But then the receiver would not play d after observing R, hence there is no such pooling PBE that fulfills signaling requirement 6.